



NFW 23

Neural fields equations, from Wilson-Cowan  
to neural engineering

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## Abstracts (alphabetical order)

**Emre Baspinar** (Institut des Neurosciences Paris-Saclay (NeuroPSI))

*A neural field model for ignition and propagation of cortical spreading depression*

Cortical spreading depression (CSD) is a wave of neuronal depolarization that slowly spreads across the cortex. It is accompanied by a disturbance in ion concentration homeostasis, followed by a prolonged neuronal silence that may last for several minutes. Similarities between the CSD propagation and the spread of migraine with visual aura lead to the hypothesis that CSD is the mechanism that evokes migraine aura. In [1], we propose a new neural field model for migraine-related CSD. The model follows the Wilson-Cowan-Amari formalism. It is based on an excitatory-inhibitory neuronal population pair which is coupled to a potassium concentration variable. The novelty of this model is that it is spatially extended to a cortical layer. Therefore, it can model both the ignition and propagation of CSD. Moreover, it controls the propagation speed via connection weights and via contribution weight of each population activity to the potassium accumulation in the extracellular matrix. The simulation results regarding the propagation speed are in coherence with the experiment results obtained from mice [2].

### References

[1] E Baspinar, D Avitabile, M Desroches, and M Mantegazza. A neural field model for ignition and propagation of cortical spreading depression. 2023.

[2] Oana Chever, Sarah Zerimech, Paolo Scalmani, Louisiane Lemaire, Lara Pizzamiglio, Alexandre Loucif, Marion Ayrault, Martin Krupa, Mathieu Desroches, Fabrice Duprat, et al. Initiation of migraine-related cortical spreading depolarization by hyperactivity of GABAergic neurons and NaV 1.1 channels. *The Journal of Clinical Investigation*, 131(21), 2021.

**Marcelo Bertalmio** (Spanish National Research Council (CSIC))

*Modeling challenging visual phenomena with Wilson-Cowan equations having locally shifting nonlinearities*

In the original formulation of the Wilson-Cowan equations, the activation function nonlinearity can be regarded as the average of single-neuron activation functions. Here we take into account more recent results from visual neuroscience and view the activation function as a dendritic nonlinearity, which can vary spatially according to the neuronal input. This apparently small modification has a profound impact in the modeling capabilities of the Wilson-Cowan equations, allowing to predict a number of very challenging perceptual and neural phenomena that remain out of reach for classical models.

**Virginia Bolelli** (Université Paris-Saclay / CentraleSupélec)

*On the neural connectivity kernel for stereo: how the geometry of the world is reflected in the geometry of connections*

In a previous study, we proposed a neurogeometrical model for stereo vision. More precisely, the functional architecture of binocular cortical mechanisms in V1, together with psychophysical experiments regarding 3D association fields, can be formalized through a sub-Riemannian structure in  $\mathbb{R}^3 \times \mathbb{S}^2$ , inspired by previous work on the stereo problem of Li-Zucker and sub-Riemannian model for monocular vision of Citti-Sarti.

This presentation will focus on the emergence of 3D perceptual units from stereo vision. Firstly, the evolution in time of the activity of the neural population, modeled through a Wilson-Cowan mean field equation, has been extended taking into account the functional architecture of the visual cortex by Bressloff-Cowan. In the binocular case, we extend this model introducing a connectivity function that can be obtained as a fundamental solution of a sub-Riemannian Fokker Planck in  $\mathbb{R}^3 \times \mathbb{S}^2$ . Secondly, following the ideas of Bressloff-Cowan for hallucination patterns, and Sarti-Citti for the emergence of bidimensional percepts, we perform stability analysis of the integro-differential equation to show the emergence of 3D perceptual units.

The work is joint research with G. Citti, A. Sarti, and S. W. Zucker.

**Lucas Brivadis** (Centre national de la recherche scientifique (CNRS))

*Adaptive observer and control of spatiotemporal delayed neural fields*

We propose an adaptive observer to asymptotically estimate the synaptic distribution between neurons from the online measurement of part of the neuronal activity and a delayed neural field evolution model. The convergence of the observer is ensured under a persistency of excitation condition. We show how it can be used to derive a feedback law ensuring asymptotic stabilization of the neural fields. Under additional restrictions that we will discuss, we propose a modification of the feedback law to ensure simultaneously practical stabilization of the neural fields and asymptotic convergence of the observer.

**Frédéric Chavane** (Institut de neurosciences de la Timone)

*Understanding mesoscopic cortical dynamics with the help of mean field models: a view from an experimentalist*

The visual system is generally described as a fast and efficient feedforward hierarchical process. However, most of the knowledge we have accumulated over decades comes from stationary visual stimuli, in the sense that the features for which the visual system is selective (position, orientation, spatial frequency, direction etc....) do not change during presentation time. Because these features are organized into topographic maps in the visual system, such stationary stimulation generates stationary activations, well explained by a feedforward process. To understand how the visual system processes non-stationary stimuli, which are most common under natural conditions, we need access to mesoscopic scales to observe the resulting dynamic activations in topographic maps. In this regard, voltage-sensitive dye imaging is the only tool that enables recording these dynamics with high spatial and temporal resolution over a wide field of view. However, the activation dynamics of the neuronal population remains a complex process whose interpretation is not trivial. In this presentation, I will show examples to argue that mean field models, due to a shared scale constraint, are the ideal computational tool to interpret and dissect the potential origin of the observed signal.

**Stephen Coombes** (University of Nottingham)

*Pattern formation in biological neural networks with rebound currents*

Waves and patterns in the brain are well known to subserve natural computation. Much attention in the theoretical neuroscience community has been devoted to analysing networks of relatively simple spiking neurons (IF type) or firing rate models (Wilson-Cowan type) and to great effect! Indeed, the understanding of how spatio-temporal patterns of neural activity may arise in the cortex has advanced significantly with the development and analysis of such models. To replicate this success for sub-cortical tissues requires an extension to include relevant ionic currents that can further shape firing response.

Here I will advocate for two complementary approaches: i) that augments the approach for IF networks to include piecewise linear caricatures of gating dynamics for nonlinear ionic current models, ii) firing rate reductions for systems where the nonlinear ionic currents are slow. By way of illustration, I will show how to construct spatially periodic waves and patterns in i) a simple spiking tissue model of medial entorhinal cortex (with an  $I_h$  current), ii) a firing rate model of thalamus (with an  $I_T$  current).

The biological commonality between these two models is that both express local 'rebound' currents that can usefully shape global tissue response. The mathematical commonality is the use of tools from non-smooth dynamical systems theory to make analytical progress in determining patterns and their stability.

**Alain Destexhe** (Institut des Neurosciences Paris-Saclay (NeuroPSI))

*A general semi-analytic mean-field formalism to model complex activity states in neural systems at different scales*

Mean-field models are an essential tool to link microscopic and macroscopic scales in the brain. This aspect is illustrated by showing a powerful Master-Equation based formalism to build mean-field models of network of neurons, with little compromise on biophysical properties. This approach was applied to various neuronal models of different levels of complexity, such as the Hodgkin-Huxley model, and different variants of Integrate-and-fire models. Because the ME formalism is entirely defined by the transfer function of the neurons, we put emphasis on the calculation of this function. We show that a general template can capture the transfer function of many neuron types, and validate the mean-field obtained with numerical simulations of the spiking network. We show applications of this formalism to simulate phenomena at the mesoscale, such as cortical propagating waves, or large scales, such as the responsiveness of different brain states to external stimuli. Finally, we show preliminary results where mean-field models can be used to predict emerging large-scale effects of alterations of synaptic receptors, with applications to model drugs or anesthetics.

**Alessio Franci** (Université de Liège)

*Fast and flexible multi-agent decision-making in recurrent neural networks*

I will introduce a tractable and principled model of multi-agent multi-option decision-making and thoroughly explore its opinion-forming bifurcation behavior. Although derived from model-independent symmetry arguments, our model can be interpreted as a continuous time recurrent neural network, including finite-dimensional Wilson-Cowan dynamics and continuous Hopfield networks as special cases. Depending on tunable and interpretable parameters, the model can exhibit rich opinion forming behaviors (including agreement, consensus, disagreement, and polarization) as an 'attention' bifurcation parameter is increased past a critical value. When attention is controlled through state-dependent feedback, the model is capable of transitioning from indecision to decision through input-driven switches in its multi-stable decision attractor landscape and of exhibiting new kinds of excitable decision-making behaviors. Decision-making organized by feedback-controlled bifurcations is fast because of the localized unstable behavior brought by bifurcations and flexible because of sensitivity, robustness, and adaptability exhibited by dynamical systems close to bifurcations. Our model provides a new framework for understanding collective behavior in biological and social systems, and for designing fast and flexible decision-making dynamics in artificial embodied agents, including neuromorphic systems. Ongoing extensions to infinite-dimensional decision-making dynamics inspired by neural fields will also be discussed.

**Axel Hutt** (Inria MIMESIS at Strasbourg)

*Additive Noise-Induced System Evolution (ANISE)*

Additive noise has been shown to tune the dynamics of nonlinear high-dimensional systems. For instance, close to bifurcation points it may advance or delay the bifurcation and hence affect the system stability. Very recently, studies of nonlinear random networks have shown that additive stochastic forces may tune the systems stability, oscillation frequency and even its degree of synchronisation similar to coherence resonance. The presentation will demonstrate these effects by a mean-field theoretic description and will show how they explain various brain activity phenomena, such as induced brain fragmentation in general anaesthesia and switch of occipital EEG-rhythms while opening/closing eyes in humans.

**Elif Köksal Ersöz** (Institut National de la Santé et de la Recherche Médicale)

*Neural mass modeling for interpretation of interictal epileptic discharges in partial epilepsies*

Interictal epileptic discharges (IEDs) are brief paroxysmal events (duration ~80-500 ms) that are observed either in the scalp (EEG) or intracerebral (SEEG) signals. These epileptiform events arise from the synchronous activation of neurons in the cerebral cortex. IEDs demonstrate a wide range of morphology. They are observed in epileptogenic and non-epileptogenic zones and may trigger seizures or prevent seizure propagation. Therefore, patient-specific modeling of IEDs can predict the location of epileptogenic zones. In this talk, I will first focus on a specific class of IEDs, so called spike-and-waves (SWs), characterized by a short-duration spike followed by a longer duration wave. I will develop a neural mass model for neocortical structures accounting for cortical layers. The model suggests that basal synaptic projections generate the spike component of an SW, while apical inhibitory projections generate the wave component. The morphological differences between epileptogenic and non-epileptogenic zone SWs appear to be due to the level of sustained inhibition. Then, I will combine physiologically relevant computational and anatomical models to reconstruct in silico SEEG signals and to link the signal morphology with the geometry of underlying epileptic regions. The overall framework extends neural mass models to mesoscale and macroscale modeling of epileptiform activity by merging neurophysiology, mathematical modeling, and biophysics to reproduce accurately recorded signals in a patient-specific manner.

#### References

- E. Köksal Ersöz, R. Lazazzera, M. Yochum, I. Merlet, J. Makhalova, B. Mercadal, R. Sanchez-Todo, G. Ruffini, F. Bartolomei, P. Benquet and F. Wendling. "Signal processing and computational modeling for interpretation of SEEG-recorded interictal epileptiform discharges in epileptogenic and non-epileptogenic zones", *J. Neural. Eng.*, 2022.
- E. Köksal Ersöz, C.-G. Bénar, P. Benquet, F. Wendling, and I. Merlet, "Refinement of granularity of cortical atlas to mode epileptic interictal spike propagation", *Neuromodulation* 25(7), S188, (2022).

**Michael Rule** (University of Cambridge)

*Experimental evidence for spatiotemporal resonance in mouse primary visual cortex*

We will present experimental evidence of standing waves in mouse visual cortex evoked by focal flickering light. These recordings were obtained via mesoscopic glutamate fluorescence imaging, and capture the mean population activity of excitatory neurons at 67 micron resolution. We report stimulus induced standing waves confined by the boundary of visual cortex, which exhibited multiple resonances reminiscent of Chladni's patterns in vibrating metal plates. At first glance, these results appear inconsistent with existing Wilson Cowan neural-field models of flicker-induced spatiotemporal pattern formation in visual areas. We will discuss the experimental data in context of available theories, highlight as-yet unresolved modelling challenges, and conjecture on the implications of these results for flicker-induced visual hallucinations and traveling waves in cortex.

**Cyprien Tamekue Woundja** (Université Paris-Saclay / CentraleSupélec)

*Mackay-type visual illusions via control of Neural fields equations*

Bifurcation theory appears to be one of the most used mathematical tools to address neuroscience questions, mainly in describing (spontaneous) pattern formation in the primary visual cortex (V1) under a sudden qualitative change of some parameter. It remains a powerful tool in understanding sensory-driven and self-organised cortical activity interactions, mainly when the sensory input is fully distributed in V1, Nicks et al. (SIAM J. Appl. Dyn. Syst., 2021). Nevertheless, in the presence of localised sensory inputs used, e.g., in the psychophysical experiments of Mackay (Nature, 1957) and Billock and Tsou (PNAS, 2007), techniques from bifurcation theory and even from multi-scale analysis seem ineffective for describing these phenomena, in particular, Billock and Tsou's experiments that we prove for being wholly nonlinear. We expound a mathematical framework to explain these phenomena in V1 (thereby in the retina due to the retino-cortical map), which consists of input-output controllability of an Amari-type neuronal fields model. In our discussion, the sensory input is interpreted as a cortical representation of the visual stimulus used in each experiment. It contains a localised distributed control function that models the stimulus's specificity, e.g., the redundant information in the centre of Mackay's funnel pattern ("Mackay rays") or the fact that visual stimuli in Billock and Tsou's experiments are localized in the visual field.

**Federico Tesler** (Institut des Neurosciences Paris-Saclay (NeuroPSI))

*Modeling brain signals with mean-field models: recent developments for LFP, MEG and fMRI signals*

Mean-field formalisms provide a relevant tool for modeling the activity of large neuronal populations and they are now playing a key role in new implementations of whole brain simulations. Nevertheless, the modeling of typical brain signals from mean-field formalisms (such as LFP, MEG and fMRI) is still an open question. These brain signals provide the experimental information about the activity of neuronal populations, for which the link between them and the mean-field models constitutes a very relevant

issue. In this talk I will present recent developments in the modeling of LFP, MEG and fMRI signals from mean-field formalisms. The calculation of LFP is done via a kernel method based on unitary LFP's (the LFP generated by a single axon) which was recently introduced for spiking-networks simulations and that we adapt here for mean-field models [1]. The calculation of the magnetic field is based on current-dipole and volume-conductor models, where the secondary currents (due to the conducting extracellular medium) are estimated using the LFP calculated via the kernel method and where the effects of medium-inhomogeneities are incorporated [1]. Finally, the calculation of the fMRI-BOLD signal is performed via a biologically plausible model of the neurovascular coupling, where a feed forward system of neurons-astrocytes-vasculature is considered, and where the calcium activity of astrocytes provides the link between the neuronal and vascular systems [2]. The methods presented in this talk have been recently published [1,2] and are currently being applied in whole brain-simulations [1].

#### References

- [1] Tesler et al., Mean-field based framework for forward modeling of LFP and MEG signals, *Frontiers in Computational Neuroscience*, 2022.
- [2] Tesler, Linne and Destexhe, Modeling the relationship between neuronal activity and the BOLD signal: contributions from astrocyte calcium dynamics, *Scientific Reports*, 2023.

**Cem Uran** (Ernst Strüngmann Institute for Neuroscience / Max Planck Society)

*Contextual modulation of neural signals in the primary visual cortex of awake macaque monkeys*

I will present our experimental findings on gamma oscillations in awake macaque V1 (see <https://pubmed.ncbi.nlm.nih.gov/35120628/>), and discuss planned future work on the E/I balance (PING) model for closed-loop control applications to investigate the neural mechanisms of visual (predictive) processing. I won't present results on Wilson-Cowan modeling but introduce our project based on it.

The dynamics of gamma oscillations are well captured by stochastic Wilson-Cowan equations. These cortical dynamics reflect a tight E/I balance. The balance between excitation and inhibition is a network regime where the membrane potential is right below the threshold allowing for highly responsive states while still keeping the overall activity in balance. Dynamical switching between spontaneous and evoked responses while keeping a balanced state together with state-dependent modulation of neural activity are key properties of information processing in the awake cortex. Gamma oscillations reflect the binding of spatiotemporal ensembles of neurons that are predictive of each other and relate to the perception of Gestalt grouping principles. On the other hand, firing rates reflect salient features that are spatiotemporally unpredictable. These complimentary aspects of network activity explain behavior and at the same time lead to the efficient and predictive processing of the sensory inputs.

**Romain Veltz** (Inria Center of University Côte d'Azur)

*Theoretical / numerical study of modulated traveling pulses in inhibition stabilized networks*

Cortical waves have been the subject of active studies. Some of the latest results concern the stimulus evoked activity in the visual cortex of awake monkeys where it was experimentally shown that this cortical activity is a traveling wave. In this talk, we informally review the results associated with the dynamics of waves in 1d neural field models. We prove a principle of linearized stability for traveling waves and we show the existence of a center manifold which allows the use of normal form / bifurcation theory. Finally, we present two numerical schemes to compute modulated travelling waves with an application to a spatialised inhibition stabilised network.